A New Model of Learning

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November 1, 2005

Abstract

In this letter we present a mathematical model of complex learning that combines the theories of Freeman [Freeman, 1999] on mesoscopic brain dynamics with the theories of Langer [Langer, 1995] on mindful learning and the empirical field studies of R. Brown and B. Brown of amateurs learning professional tennis skills. The significance of the Freeman-Langer model is that it bridges the dynamics of neurons with the dynamics of human behavior.

Sherrington has suggested that locomotion involves a modulation of postural reflexes which are assumed to be "elementary" or "primitive" in some genetic sense and that these elementary reflexes can be assembled or integrated to form complex purposeful actions. The formation of these "mesoscopic" assemblies is a process that is driven by instruction of the student, observation by the student of other subjects and internal reflection by the student. It is our objective to offer the first mathematical model of the dynamics of the mesoscopic assembly process.

There many factors that affect the assembly formation process, but in the interest of forming an initial model we will only consider four. The first factor we call capacity. There is an "optimal" neural assembly size for any given component and as the development of this component approaches this size, the mean assembly size exponentially converges toward constant value.. The second factor is the cyclic nature of assembly formation. This factor accounts for the fact that assemblies grow and regress according to a nonlinear cycle that is not fully understood at this time. The third factor is called regression and represents the tendency of a system to return to a previous stable state. The fourth process is the compatibility between the receiving process of an individual brain and the input process offered as instruction, observation or reflection.

1 Introduction

Our model is derived from three sources: Freeman's theory of mesoscopic brain dynamics [Freeman, 1999], Langer's psychological model of the human learning process summarized in [Langer, 1996] and ten years of field observations of amateurs learning professional tennis skills. By combining the theory of Freeman with the theory of Langer we have a mechanism to bridge the gap between the dynamics of neurons and dynamics of observable "every day" human behavior. The synthesis of these two theories provided the necessary framework within which to conduct field observations.

1.1 Field Observations of the Complex Learning Process

Initially a student is shown a stroke (a demonstration) combined with a verbal explanation. From this the student's initial attempts can be quite good. However, within one or two days this initial "skill" vanishes and the entire process must be repeated. On the second repetition, progress is not so successful as on the first occasion. Possibly due to the student trying to remember what they did. As there is no memory of their first efforts, this is futile.

Here is where the learning process begins in earnest. The instruction is repeated. And the student begins to form a first approximation skill in only a few minutes. The student then attempts to improve this action sequence as the instructor feeds them balls. Then something dramatic occurs. Without explanation, the student begins to demonstrate wild variations on the instruction (this is explained by Langer's theories. They do this in spite of their desire to continue to follow instructions. This is a source of endless frustration to the student as they cannot understand why their body is reacting in this odd manner against their will. The outcome of this session is a "draft" first approximation of a stroke.

At this point two directions have been followed. One is to teach the student a full integrated professional (complex) stroke; the other is to teach the student stroke components. Stroke components are individual actions, which, when executed in series, form a stroke. The action components are derived from a process in which the full stroke is divided into smaller components which are as closely aligned to primitive reflexes as possible and for which a relevant purpose can be articulated. The Freeman-Langer theory suggests that the ladder approach results in very rapid learning as compared to the first and this is confirmed by our field experiments.

In both approaches, the student forms a first approximation that is crude, but which provides an essential foundation for the formation of a second approximation based on further instruction. However, the Freeman-Langer theory provides an order of magnitude improvement in learning speed. Hence our model is organized around their theory.

When instruction is purposely made crude and ambiguous it is found that the first approximations rapidly decay and the initial skill is lost. In many cases, no information is transfered at all. Still in other cases, instruction may be presented by one instructor with a failure to form a first approximation and then the same instruction presented by another instructor on a widely separated occasion at which time a successful first approximation is formed. This occurs at all levels of instruction, first, second, and nth approximations.

A further empirical observation is called the *Principle of Imperfection* whereby the student, if they learn the "perfect" stroke, never consistently executes this stroke, but executes variations around this "perfect" stroke, with the brain constantly testing new possibilities of actions to see if an improvement is possible. This, we conjecture, is a never ending process intrinsic to human learning dynamics.

Overnight or over a period of days that varies with the subject, the draft first approximation is improved to form the first approximation. Individual preconceptions can lead to a substandard formations in which case the initial formations will eventually be totally disassembled, resulting in no progress in the learning process. However, even though an approximation may be disassembled, some remnants of the assembly may survive. If the draft first approximation is consistent with existing assembles, the draft approximation will be refined by neural pruning to produce a consistent stable first approximation of the objective action that will persist for several days. Following the successful formation of a first approximation, the subject is prepared to continue the learning exercise.

1.2 Other Empirical Observations

Other empirical field observations that support the Freeman-Langer theory are as follows: (1) Complex actions learned from protocols designed from the Freeman-Langer theory are more adaptable than the same action designed around rote learning protocols. (2) Given a learned action or concept is in place in the brain, a new action or concept that has components that are contradictory to the existing concept will either be disassembled, or will cause the existing structure to be disassembled or degraded. In terms of macroscopic human activity, the individual will either be unable to retain knowledge of the new action or concept or the performance of the existing action or use of the existing concept will be measurably degraded. (3) The Freeman-Langer theory, when applied to conceptual learning, facilitate the formation of analogies as a basic mechanism of human thought. Hence, subjects who learn a complex activity or concept using instructional protocols derived from the Freeman-Langer theory will be able to form analogies faster and more frequently than subjects who learn a complex activity or concept through other learning protocols. This fact is reflected in players being able to form dynamical analogies during a match. (4) If during a training process the correct mesoscopic assemblies are formed, they will be reused in new situations. As a result, training in one area can result is learning in an area in which no training has been previously provided. (5) Action sequences are efficiently organized starting from an objective. The assembly of an action sequence stimulated by a purpose proceeds by selecting components in the reverse order of their execution. (6) Template actions formed through a rote process will be disassembled with only their generic parts retained. (7) Mesoscopic components that are invariants of numerous action sequences are retained with a higher probability that components that are infrequently used. (8) The learning process is indistinguishable from a rehabilitation process. It is impossible to perform an experiment to clearly distinguish between a learning process of a normal subject and the rehabilitation process of a trauma subject.

As a final note, we are making a distinction between learning a complex skill and learning a simple skill. The distinction is this. Simple skills only involve the modulation of "primitive" movements such as walking or riding a bicycle. The development of these skills appears to proceed in a manner quite separate from developing a skill to serve or hit a professional forehand in tennis. For example, from day to day the walking function appears invariant once learned. This is not true of complex skills such as found in tennis strokes. There are volumes of evidence supporting the assertion that one can learn to serve quite well on one day and be totally unable to serve the following day.

2 The Model

In our model, see Eq.(1) below, x is the size of the mesoscopic assembly in neurons. The input (stimulation) process, $f(t)$ is a chain of positive impulses. In order to model relevance based versus rote learning [Langer, 1996] there must be a correlation factor, $k(t)$, between the input, $f(t)$, and acceptance process, $g(t)$, that affects formation or decay of assemblies. The time interval between impulses may vary in rote or complex ways. The parameter β in the equation is experimentally determined. The form of the function $h(u, v, w)$ may vary and methods of modeling it are treated in the Appendix. Thus our learning equation is:

$$\frac{d^2x}{dt^2} + (\beta(1-k(t)))\frac{dx}{dt} + h(x, \frac{dx}{dt}, t) = \int_0^t f(s)ds \tag{1}$$

where

$$k(t) = \frac{\int_0^t f(s)g(s)ds}{(\int_0^t f(s)ds \int_0^t g(s)ds)^{1/2}}$$

To emphasize the relationship between the input function and the acceptance function we rewrite the equations as follows:

$$\frac{d^2x}{dt^2} + h(x, \frac{dx}{dt}, t) = \int_0^t f(s)ds - \beta((1-k(t)))\frac{dx}{dt} \tag{2}$$

Several cases arise depending on the dynamics of the expression:

$$\int_0^t f(s)ds - \beta(1-k(t)\frac{dx}{dt} \tag{3}$$

The baseline equation is given by:

$$\frac{d^2x}{dt^2} + h(x, \frac{dx}{dt}, t) = 0 \tag{4}$$

It is trivial to derive second order ODE's to model chaotic processes as is demonstrated by various publications by Brown and Chua [1993,1996,1997,1998,1999,2001] etc. A simple example is the forced Duffing equation. Equation (4), for purposes of simplification, may be considered as the baseline state of a mesoscopic assembly which is an n^{th} approximation in a learning process. In the baseline case, the neural assembly size will vary chaotically around a mean value over time. However, without purposeful experience [Langer, 1996], it will decay. This fact is not accounted for in the baseline model but will be the subject of a forth coming paper. By adding stimulation, $f(t)$, the model must be enlarged as seen in Eq. (1). When the input is highly correlated with the acceptance function, the damping factor will be minimal and the resulting assembly will grow in size, then stabilize and then fluctuate around a mean value in a chaotic fashion. This is the ideal learning process theorized by Langer and supported by field observations. When acceptance and input are uncorrelated, the assembly may decay rapidly. Interesting questions arise related to the form of the acceptance function, $g(t)$, for a normal human. Most evidence suggests that this is a chaotic function. Further, it is most likely that this function has the form $g(t, x, \dot{x})$. When Eq.(3) is chaotic, the normal mode, the dynamics of x resemble a stochastic process.

EXAMPLE: In this example we use the forced Duffing equation because of its wide familiarity. It is not suggested that this is a typical model for mesoscopic assembly dynamics:

$$\frac{d^2x}{dt^2} + (\beta(1-k(t)))\frac{dx}{dt} + \alpha\frac{dx}{dt} = \int_0^t f(s)ds + 7.5\cos(t) \tag{5}$$

Rearranging this equation we have:

$$\frac{d^2x}{dt^2} = -((\beta(1-k(t))) + \alpha)\frac{dx}{dt} + \int_0^t f(s)ds + 7.5\cos(t) \tag{6}$$

The function $f(t)$ can be modeled as

$$\gamma\exp(-50(\cos^2(\omega(t-a))))$$

An example time series from Eq.(**??**) is given in Fig. 1.

FIGURE 1

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In Fig. 1, the green graph is the size of the mesoscopic assembly as a function of time. The red graph is the phase plane plot of size versus rate of change in size. For this figure $\gamma = 25.0, \alpha = 0.05, \omega = 1/(96\pi), a = 40.0$. As seen in the figure, as each increment of 'perfect information' is provided by the impulse function, the state of the assembly changes, becoming larger, with lower variance. This is in agreement with field observations. The red graph is a perturbation of the Duffing phase plane plot, with the plot's width increasing in a positive direction with each impulse increment.

3 Circuits

A circuit for the global learning model is seen in Fig.2. The nonlinear elements are constructed from IC's.

FIGURE 2

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Corresponding to the circuit diagram is a standard control schema, Fig.3. When two competing assemblies are related, the scheme of Fig.4 suggests how each assembly affects the other. As the number of neurons can be considered as limited, the acquisition of a neuron by one assembly results in the loss of a neuron by another assembly. The phenomena of encroachment is well documented and competition for neurons by assemblies is a variant of this process. In the simplest global model, the acquisition of neurons by one assembly from another can be modeled by increasing the resistance of the growth of the first assembly by the instanteous growth of the second assembly. This is illustrated in Fig.4. The stability of this process will be investigated in a later paper and is a key to the phenomena of self organization.

4 Summary

The Freeman process of the formation and integration of mesoscopic assemblies can be directly related to the mindful learning process of Langer providing a bridge between neurodynamics and psychology. We refer to this as the Freeman-Langer Bridge. There are numerous benefits that arise from this model. The most important benefit is an understanding of how neurodynamics is reflected in human behavior. For example, the common place phenomena of learning a complex skill only to have it quickly "disappear" is explained by the disintegration of templates. On the other hand, the development of a skill from the development of the "right" components accelerates related skill development.

The significance of the Freeman-Langer Bridge for the learning, and thus training process is clear. However, the importance to robotics is can be explained as well. For example, for a bot to learn, then its design must be guided by the Freeman-Langer Bridge. In particular, rule based learning corresponds to template based learning in humans. We know this form of learning is not adaptable to new circumstances because the stimulation of the use of template results in a run-to-complete process. This form of activity is seen in animals and can be confused with learning, but clearly is not complex learning. By designing a Bot from fundamental action components and providing the Bot with an assembly process, the Bot will have the ability to adapt.

The relationship between the Freeman-Langer Bridge and neural and Bayesian networks deserves mention. The Freeman-Langer bridge is a dynamical model whereas neural and Bayesian nets are data models. The simplest analogy is that of using a least squares approach to fitting a data set versus using a differential equation to fit the dynamics of the data. Dynamical models carry for more powerful predictive information than curve fitting processes which must be reevaluated when new data arrives.

5 Appendix:Models of Nonlinearities

In order to model the nonlinear component of the learning process expresses by $h(u, v)$ we may use mathematical tools developed by Brown [Brown & Chua, 1998] In this section we review some of these tools.

From [Brown and Chua, 1998] we recall that all two-dimensional vector fields can be put into the form:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \left(\frac{\dot{r}}{r}\mathbf{I} + \dot{\theta}\mathbf{B} \right) \begin{pmatrix} x \\ y \end{pmatrix}$$

where $\mathbf{I}$ is the identity matrix and $\mathbf{B}$ is the matrix

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

When $\dot{r} = 0$ we get the equation:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \left(\dot{\theta}\mathbf{B}\right) \begin{pmatrix} x \\ y \end{pmatrix}$$

the orbits must be circles, the same as the simple harmonic oscillator and the twist equation. For $\dot{\theta} = 1$ we obtain the simple harmonic oscillator. For $\dot{\theta} = r$ we obtain the twist. However, if

$$\frac{\partial \dot{\theta}}{\partial r} = 0$$

we are still in a position to obtain closed-form solutions. For example, if $\dot{\theta} = \sqrt{1 - k^2 \sin^2(\theta)}$ we obtain the closed-form solution

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} r\,\mathrm{cn}(t + C) \\ r\,\mathrm{sn}(t + C) \end{pmatrix}$$

where sn, cn are the Jacobi elliptic functions.

Since $\dot{\theta}$ is not a function of r, the angular velocity does not change from orbit to orbit. Such a system preserves lines through the origin and through any complete revolution a line or a region is mapped onto itself. The source of the nonlinearity is that along an orbit, the arc length is expanded and contracted in a periodic manner. In this system, matter is neither created, as happens in systems having a source, nor destroyed, as happens with systems having a sink, but rather is alternately compressed and stretched.

Chaotic electronic circuits of this system can be designed by the standard two-phase gate method: If we translate the system to $(a, 0)$ and compose it with the flip we obtain a Poincaré map that produces chaos while having only periodic hyperbolic points and no hyperbolic fixed points. The origin of chaos in a system of this type is solely from the nonlinear acceleration taking place around circles. In our learning model, the nonlinear change in the "learning cycle" can be captured by modeling $\dot{\theta}$.

In our model $\dot{r}$ describes the regression dynamics of learning.

In order to expand the our modeling tools we use the form of a vector field mentioned earlier:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \left(\frac{\dot{r}}{r} \mathbf{I} + \dot{\theta} \mathbf{B} \right) \begin{pmatrix} x \\ y \end{pmatrix}$$

and make two assumptions. The first is that its underling group is measure-preserving or, what is the same thing, the vector field is divergence-free. The second is that the system preserves lines through the origin. Using these two assumptions we derive the following partial differential equation for $\dot{r}$:

$$\frac{1}{r} < \mathrm{X}, \nabla \dot{r} > + \frac{\dot{r}}{r} + < \mathbf{B}\mathrm{X}, \nabla \dot{\theta} >= 0$$

With the following notational convention we obtain the PDE in standard form. Let

$$\begin{pmatrix} \dot{r} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} g(x, y) \\ f(\theta) \end{pmatrix}$$

Now the PDE becomes

$$x\,p + y\,q = -(z + r\,f'(\theta)) \tag{7}$$

where $p = z_x$ $q = z_y, z = \dot{r} = g(x, y)$. The general solution is given by

$$z = h(x, y) F(x/y)$$

where h is dependent on $f'(\theta)$. If we assume that $c\,z = r\,f'(\theta)$ is the form of the solution, then we obtain the following consistency equation to check:

$$x\,p + y\,q = -(c + 1) z \tag{8}$$

For simplicity we require that $-(c + 1) > 0$ or that $c < -1$. All of these assumptions would be fine if the solution of the resulting equation is consistent with these assumptions. By an application of standard methods for solving first-order partial differential equations we get

$$z = \frac{y}{-(c + 1)} F(x/y)$$

The consistency check we must make is to see whether

$$r\, f'(\theta) = \frac{y}{-(c+1)} F(x/y)$$

is possible. Since $y = r\,\sin(\theta), x = r\,\cos(\theta)$ we see that if we choose $f(\theta) = a + b\sin(\theta)$, everything is consistent. In particular, we have

$$r = r_0 \left(\frac{f(\theta_0)}{f(\theta)}\right)^{1/c}$$

and the first part of the solution is done. Now, if we choose $a > b$, the equation $\dot{\theta} = a + b\sin(\theta)$ is solvable in closed form for $\sin(\theta)$. Using a standard table of integrals we get

$$\frac{b + a\sin(\theta)}{a + b\sin(\theta)} = \sin(k\,t + C_0)$$

where $k = \sqrt{a^2 - b^2}$, and C_0 is a constant of integration to be determined from the initial conditions. From this relation we obtain $\sin(\theta), \cos(\theta)$ and we are done.

The general solution in rectangular coordinates is given by:

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = r_0 \left(\frac{f(\theta_0)}{f(\theta)}\right)^{1/c} \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix}$$

where we must have $c > 1$. Note that the root factor is not a constant since $f(\theta)$ is a function of time. The orbits cannot be linear and, by construction, the system is divergence-free. Using the two-phase gate method, we may make this map a component of a Poincaré map of a nonlinear circuit.

It is likely that brain dynamics may have nonzero divergence. It is possible to obtain nonzero divergence equations that are just as useful. One option is to solve the PDE $x\,p + y\,q = z$ and the choice

$$\dot{r} = -r f'(\theta)$$

with $\dot{\theta} = f(\theta) \neq$ constant gives the closed-form solutions in rectangular coordinates:

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = r_0 \frac{f(\theta_0)}{f(\theta)} \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix}$$

Note that if $\dot{\theta} = -1$ and $\dot{r} \neq 0$ we also get nonzero divergence.

This process can be greatly generalized. If $\dot{\theta} = f(\theta)$ and $r = C_0 G(\theta)$ we get an autonomous ODE:

$$\dot{r} = C_0 G\text{"}(\theta) f(\theta)$$

where C_0 is eliminated from this equation by noting that $C_0 = r/G(\theta)$. So long as $\dot{\theta} = f(\theta)$ is solvable in closed form, we are done! For example, choose $f(\theta) = 2 - \sin^2(\theta)$. By use of a table of integrals we find that we can solve this equation for $\sin(\theta)$, which is all that is necessary to express the solution in rectangular coordinates. By choosing $G(\theta) = \sqrt{f(\theta)}(1 - 0.95\,\sin(\sin(\theta))$ nonlinear orbits.

This system, like the Jacobi Equation, when composed with linear maps by the method of two-phase gates can generate local attracting fixed points along side periodic, quasi-periodic, and chaotic orbits which are not attracting. The yellow orbits are chaos, the red are orbits being attracted to the period-three points. The light-blue are orbits of transient chaos that also converge to the period-three points. The dark-blue orbits are elliptic, hence represent almost periodic solutions of the ODE. Near the small yellow orbits we find chaotic, almost-periodic, and transient-chaotic solutions coexisting.

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List of Figure Captions

Figure 1: Global Dynamics of Assembly

Figure 2: Notional Circuit: Nonlinear Elements will be Resolved as IC's

Figure 3: Control Diagram

Figure 4: Notional Coupling between Assemblies